Graphene and quantum electrodynamics in 2+1 dimensions

Prof. Valery Gusynin

Preliminary knowledge:

- Dirac-Weyl equation.
- Solving Dirac equation for hydrogen atom.
- Screening of the Coulomb potential by the vacuum polarization effects.

Preliminary exercises:

- Solving the Dirac equation for the Coulomb potential in $2+1$ - dimensions.

$$[-\hbar v_F \left(i\sigma_1\partial_x + i\sigma_2\partial_y\right) + \sigma_3\Delta + V(r)]\,\Psi(\mathbf{r}) = E\Psi(\mathbf{r}),$$

with the regularized Coulomb potential

$$V(r) = -\frac{Ze^2}{\kappa r}\theta(r-R) - \frac{Ze^2}{\kappa R}\theta(R-r).$$

Here σ_i are the Pauli matrices. To separate an angular dependence use the following anzats for the spinor

$$\Psi(\mathbf{r}) = \frac{1}{r}\begin{pmatrix} e^{i\phi(j-1/2)}a(r) \\ e^{i\phi(j+1/2)}b(r) \end{pmatrix},$$

where j is the total angular momentum.

Squeezed Light Generation and Application

Prof. Dr. Boris Hage

Preliminary knowledge:

- Concept of quantum optical modes of the EM-field.
- Field quadratures and their analogy to position and momentum.
- EM-waves in linear and nonlinear ($\chi^{(2)}$) media.
- Two/many path optical interference.

Preliminary exercises:

The interaction of the fundamental mode (addressed by $\hat{a}$ and $\hat{a}^\dagger$) and the second harmonic mode (addressed by $\hat{b}$ and $\hat{b}^\dagger$) for the optical nonlinear processes of second harmonic generation or degenerate parametric amplification (OPA) assuming perfect phase matching can be described by the Hamiltonian $\hat{H}_{\text{int}} = \hbar\kappa\hat{b}^\dagger\hat{a}^2 + h.c.$, with κ characterising the strength of the nonlinear interaction.

- Discuss the key parts of this statement and the constituents of the Hamiltonian phenomenologically. *Hint: If you think this task is simple and quick, probably you're doing it wrong.*
- Justify how the squeezing operator $\hat{S}(\zeta) = \exp\left[\frac{1}{2}\left(\zeta^*\hat{a}^2 - \zeta(\hat{a}^\dagger)^2\right)\right]$ with $\zeta^* = -2\mathrm{i}\kappa\beta^* t_0$ arises from the OPA Hamiltonian acting for the duration t_0 under the condition of the second harmonic mode being in a coherent state $\hat{b}|\beta\rangle = \beta|\beta\rangle$, and $|\beta|^2 \gg \langle\hat{a}^\dagger\hat{a}\rangle$, and $|\beta|^2 \gg 1$. *Hint: Neglect the quantum character of a field, when the quantum fluctuations are small compared with the expectation value. I.e.:* $|\beta| \gg \sqrt{\langle(\hat{\delta b} + \hat{\delta b}^\dagger)^2\rangle}$ *with* $\hat{b} = \beta(\hat{1}) + \hat{\delta b}$ *and* $\beta = \langle\hat{b}\rangle$.

Electronic band structure and high temperature superconductivity

Alexander Kordyuk

Preliminary knowledge:

- Heisenberg's uncertainty principle and Pauli exclusion principle.
- Bloch wave, Brillouin zone, Fermi surface, topological Lifshitz transition.
- Peierls transition, charge and spin density waves, Fermi surface nesting.
- LandauFermi liquid, Green's function, self-energy and Dyson equation, Lindhard function (electronic susceptibility).

Preliminary exercises:

- Derive density of states (DOS) of free-electron gas for 1D, 2D, and 3D cases.
- Why the best metals (with lowest resistivity) are bad superconductors?
- Derive dependence of the imaginary part of self-energy (quasiparticle scattering rate) on binding energy for electron-electron and electron-phonon (single phonon mode) interaction.

Quantum theory of light in dielectrics — from linear optics to boson sampling

Prof. Stefan Scheel

Preliminary knowledge:

- Electromagnetic waves, mode expansion
- Canonical quantization, quantum harmonic oscillator, bosonic operator algebra
- Matrix invariants, determinants and permanents

Preliminary exercises:

- Assume that two indistinguishable photons prepared in single-photon Fock states impinge on a lossless beam splitter with reflection and transmission coefficients R and T, i.e. the state at the input of the beam splitter is $|\psi_{\text{in}}\rangle = |1_1, 1_2\rangle = \hat{a}^\dagger_{\text{in},1}\hat{a}^\dagger_{\text{in},2}|0_1, 0_2\rangle$. Given that the beam splitter transforms the photonic amplitude operators as

$$\begin{pmatrix} \hat{a}_{\text{out},1} \\ \hat{a}_{\text{out},2} \end{pmatrix} = \begin{pmatrix} T & R \\ -R^* & T^* \end{pmatrix} \begin{pmatrix} \hat{a}_{\text{in},1} \\ \hat{a}_{\text{in},2} \end{pmatrix} ,$$

  what is the probability of detecting one photon in each output arm of the beam splitter?

- The action of a lossless beam splitter can equivalently be described by the unitary operator

$$\hat{U} = T^{\hat{n}_1} \exp\left(-R^* \hat{a}^\dagger_2 \hat{a}_1\right) \exp\left(R \hat{a}^\dagger_1 \hat{a}_2\right) T^{-\hat{n}_2} .$$

  Show that the probability amplitude $\langle 1_1, 1_2|\hat{U}|1_1, 1_2\rangle$ equals the permanent of the beam splitter matrix from above. Hint: Use $\text{per}\mathbf{A} = a_{11}a_{22} + a_{12}a_{21}$ and $|T|^2 + |R|^2 = 1$.

QUANTUM COMPUTATION FROM THE PHYSICIST VIEWPOINT

Sergey N . Shevchenko

Preliminary knowledge from quantum mechanics:

- Pauli matrices for spin 1/2 particles
- Bra-ket notations
- Superposition of states

Preliminary exercises:

- Let us define the basic vectors as $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Describe the action of the spin matrices $\sigma_{x,y,z}$ on the basic states:

$$\sigma_x |0\rangle = |1\rangle , \tag{1}$$

  etc.

- Analogously, describe what would make the matrix

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{2}$$

  with the basic states $|0\rangle$ and $|1\rangle$ and also with the arbitrary state

$$\alpha |0\rangle + \beta |1\rangle . \tag{3}$$

I'll tell you that Eq. (3) describes a qubit wave function, while matrices in Eqs. (1,2) relate to the qubit operations.

Classical and quantum optics in waveguide arrays

Prof. Alexander Szameit

Preliminary knowledge:

- Tight binding model in solid state physics
- Band structures of periodic systems
- Waveguide optics

Preliminary exercises:

- Derive from Maxwell's equations the Helmholtz equation.
- Derive from the Helmholtz equation the paraxial Helmholtz equation.
- Derive the coupled mode equations from the paraxial Helmholtz equation.

Measurement Theory in Quantum Optics

Prof. Werner Vogel

Preliminary knowledge:

- Quantization of the electromagnetic field.
- Basic concepts of quantum measurements, probability distributions of observables.
- Density operator description of pure and mixed quantum states.

Preliminary exercises:

- Coherent states $|\alpha\rangle$ of the harmonic oscillator are the eigenstates of the non-Hermitian annihilation operator $\hat{a}$,

  $$\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle\,.$$

  Expand the coherent state in terms of photon number states, $|n\rangle : \ \hat{a}^\dagger\hat{a}\,|n\rangle = n\,|n\rangle$, as

  $$|\alpha\rangle = \sum_{n=0}^{+\infty} C_n\,|n\rangle$$

  and derive the coefficients C_n. Calculate the photon number probablity distribution of a coherent state.

- Derive a representation of general photon number states $|n\rangle$ as a superposition of coherent states on a circle.
- What is the minimal number of superimposed coherent states to represent the single photon state, $|n = 1\rangle$?

Nonlinear wave phenomena in Josephson junctions

Dr. Yaroslav Zolotaryuk

Preliminary knowledge:

- Quantum mechanics: two-level systems, electron in external magnetic field, magnetic flux quantization.
- Electrodynamics: Maxwell equations, inductance, self-inductance, magnetic flux, Kirchhoff laws.
- Classical mechanics: Nonlinear oscillator, elliptic functions.

Preliminary exercises:

1. Draw the phase portraits for the nonlinear oscillators $\ddot{x} \pm \sin x = 0$.
2. Consider the two-level time-dependent quantum system with Hamiltonian $\hat{H}$ with eigenfunctions $\psi_{1,2}$ and respective energies $E_{1,2}$. From the time-dependent Schrödinger equation write down the equations of motion for the probability amplitudes in the expansion $\Psi(t) = \sum_{n=1,2} C_n(t)\psi_n$.